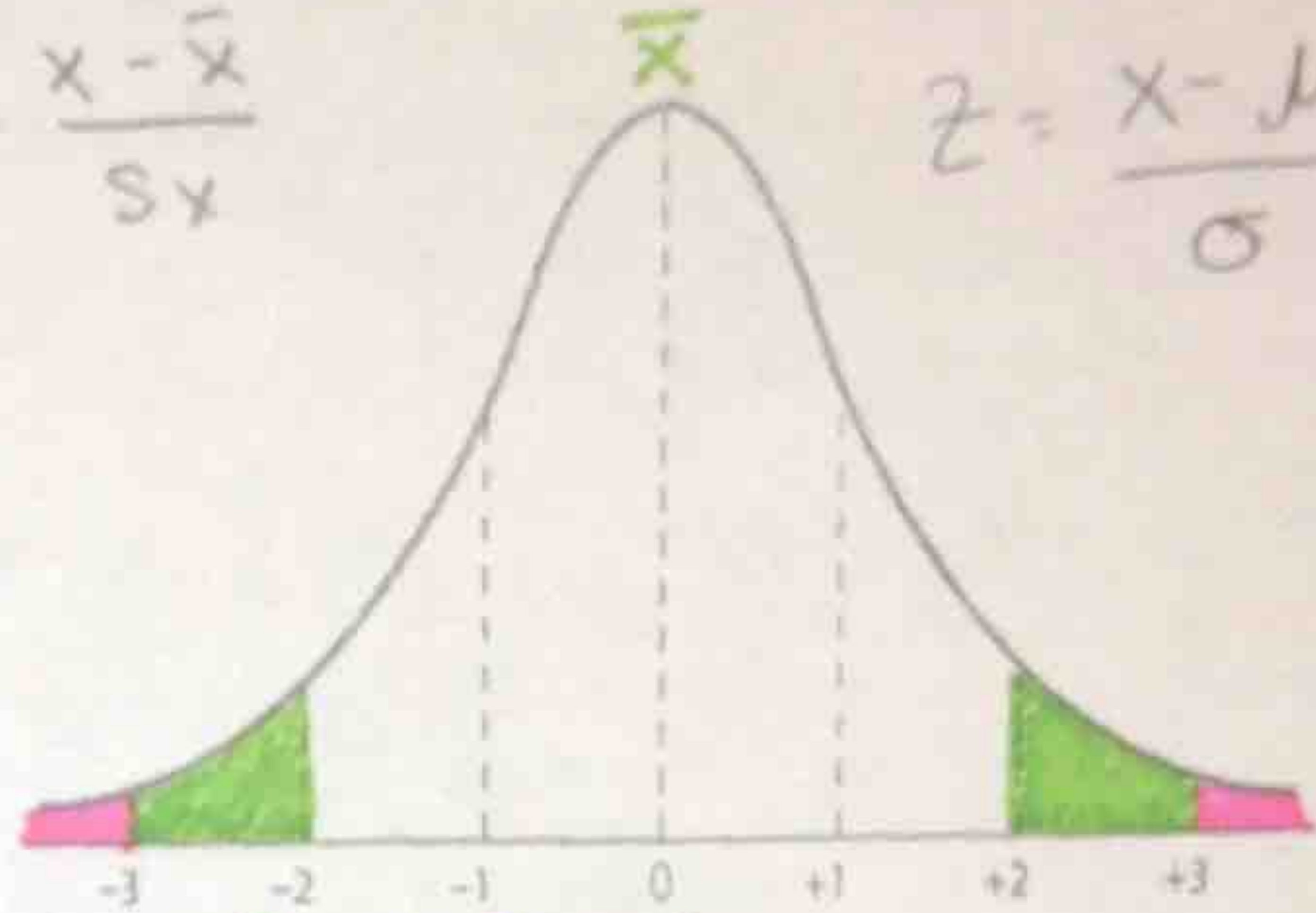
9.3 Z-Scores

SWBAT use the z-scores of a data set to find the percentage of data under the normal curve.

Z-Score: A statistical measurement of a score's relationship to the mean of a group of scores. A Z-score of 0 means the score is the same as the mean. A Z-score can also be positive or negative, indicating whether it is above or below the mean and by how many standard deviations.

$$Z\ Score = \frac{Raw\ Score - Mean}{Standard\ Deviation}$$

$z = \frac{x - \bar{x}}{Sx}$ $\qquad$ $z = \frac{x - \mu}{\sigma}$



* unusual
* Very unusual

Symbols ☺	Sample Population	Entire Population
Mean	$\bar{x}$	μ mu
Standard Deviation	Sx	σ sigma

Example 1: Suppose a normal distribution has a mean of 10 and a standard deviation of 2. Find the z-scores of the following measurements:

a) 9 $\qquad$ b) 10 $\qquad$ c) 11 $\qquad$ d) 14

a) $z = \frac{9-10}{2} = -\frac{1}{2}$ — 0.5 SD below the mean

b) $z = \frac{10-10}{2} = 0$ — 10 = mean

c) $z = \frac{11-10}{2} = \frac{1}{2}$ — 0.5 SD above the mean

d) $z = \frac{14-10}{2} = 2$ — 2 SD above the mean

Example 2: Suppose a data set is represented by a normal distribution with a mean of 125 and a standard deviation of 7.

a) What data value is 2 standard deviations above the mean?

$z = 2$ $\qquad$ $2 = \frac{x-125}{7}$ $\qquad$ $14 = x - 125$ $\qquad$ $x = 139$ $\qquad$ score = 139

b) What data value is 3 standard deviations below the mean?

$z = 3$ $\qquad$ $-3 = \frac{x-125}{7}$ $\qquad$ $-21 = x - 125$ $\qquad$ $x = 104$ $\qquad$ Score = 104

Example 3: In a normally distributed data set, find the value of the standard deviation if the following additional information is given.

a) The mean is 22.6 and the z-score for a data value of 230 is 0.2.

$0.2 = \frac{230 - 22.6}{\sigma}$ $\qquad$ $\sigma = \frac{207.4}{0.2}$ $\qquad$ $\sigma = 1037$

b) The mean is 9.8 and a z-score for the data value of 10.3 is 2.

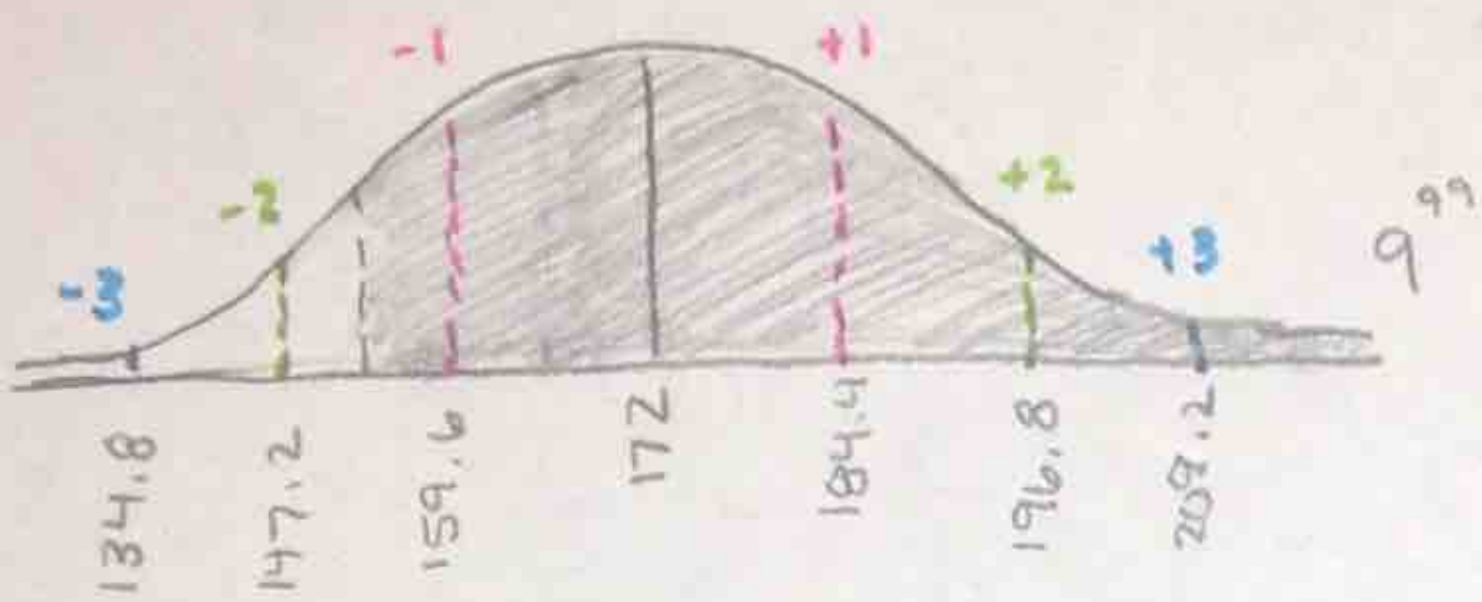
$2 = \frac{10.3 - 9.8}{\sigma}$ $\qquad$ $\sigma = \frac{0.5}{2}$ $\qquad$ $\sigma = 0.25$

ALWAYS SKETCH A NORMAL CURVE AND LABEL THE X-AXIS.

Example 1: A forest products company claims that the amount of usable lumber in its harvested trees averages 172 cubic feet and has a standard deviation of 12.4 cubic feet. Assume that these amounts have approximately a normal distribution.

How to find Proportions with Z-Scores	
Step 1:	Draw a normal curve and label the x-axis.
Step 2:	Find the z-score corresponding to the value indicated in the question. Interpret this in terms of standard deviations away from the mean.
Step 3:	Hit 2nd-VARS-2: normalcdf(lower bound, upper bound). The answer is the proportion corresponding to your z-score. Write as a percentage. **Note:** *Use 9^{99} for the upper bound if it is not already given.* **Note:** *Use -9^{99} for the lower bound it if is not already given.*

a) What proportion of trees contains more than 150 cubic feet?



$z = \frac{150-172}{12.4} = -1.77$

Normal cdf$(-1.77, 9^{99})$

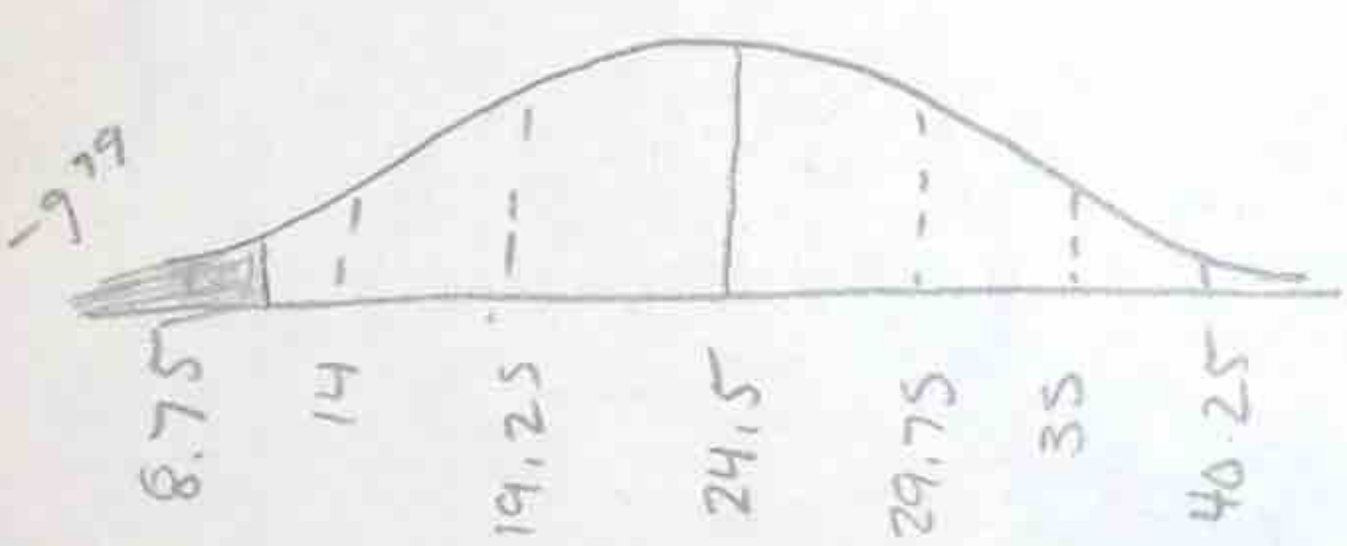
$= 96.2\%$

b) What proportion of trees contain between 175 and 190 cubic feet?

$z_L = \frac{175-172}{12.4} = 0.24$ $z_U = \frac{190-172}{12.4} = 1.45$ Normal cdf(0.24, 1.45)

$= 33.2\%$

Example 2: Healthy 10-week-old domesticated kittens have average weight 24.5 oz. with a standard deviation of 5.25 oz. The distribution is approximately normal. A kitten is designated as dangerously underweight when, at 10 weeks, it weighs less than 10.0 oz. What proportion of healthy kittens will designate as dangerously underweight?

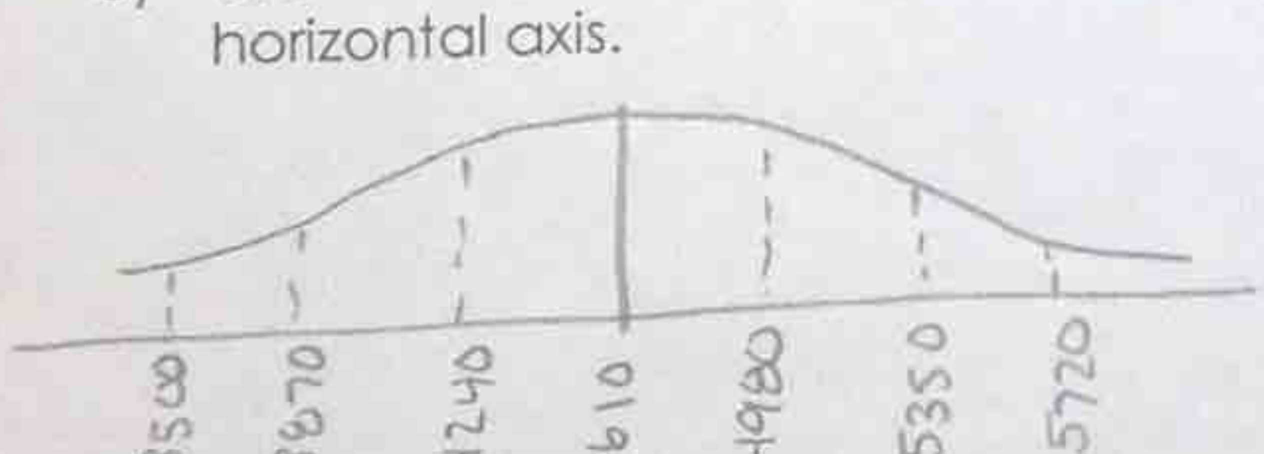


$z_U = \frac{10-24.5}{5.25} = -2.76$

Normalcdf $(-9^{99}, -2.76) = 0.29\%$

You Try! Suppose that there are 100 franchises of Betty's Boutique in similar shopping malls across America. The gross Saturday sales of these boutiques are approximately normally distributed with a mean of \$4610 and a standard deviation of \$370.

a) Draw a normal curve below and label the horizontal axis.



b) Find the z-scores of each of the following gross Saturday sales amounts: \$3870, \$4425, and \$5535.

$z = \frac{3870-4610}{370} = -2$ $z = \frac{5535-4610}{370} = 2.5$

$z = \frac{4425-4610}{370} = -0.5$

c) What percentage of Betty's Boutique franchises had gross Saturday sales between \$4425 and \$5535? Use the z-scores you found in part (a) and the normalcdf(function in the calculator.

Normalcdf$(-0.5, 2.5) = 68.5\%$

d) What percentage had gross Saturday sales between \$3870 and \$5535? Use the z-scores you found in part (a) and normalcdf function in the calculator.

Normalcdf$(-2, 2.5) = 97.1\%$

e) What percentage of stores had gross Saturday sales less than \$5535?

Normal cdf$(-9^{\wedge}99, 2.5) = 99.4\%$